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# Recursive Estimation Algorithms for Power Controls of Wireless Communication Networks

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**Abstract:** Power control problems for wireless communication networks are investigated in DS/CDMA channels. It is shown that the underlying problem can be formulated as a constrained optimization problem in a stochastic framework. For effective solutions to this optimization problem in real time, recursive algorithms of stochastic approximation type are developed that can solve the problem with unknown system components. Under broad conditions, convergence of the algorithms is established by using weak convergence methods.

**Keywords:** recursive estimation, power control, DS/CDMA, stochastic approximation, constrained optimization.

## 1 Introduction

This work is concerned with power controls for wireless communication networks, on a platform of code-division multiple access (CDMA) channels. The power control problem is formulated as a constrained stochastic optimization problem. In multi-access communication channels, such as satellite networks and wireless systems, a communication channel is shared by many users from different locations [12]. This is achieved by divisions in frequency, time, space, or combination of them. Sharing a communication channel by multiusers simultaneously implies that transmissions from the users overlap in time, frequency, and space, depending on protocols, causing potential signal interference and reduction in communication reliability and security. CDMA is a widely used protocol of multiuser communications that employ both time and frequency divisions. By spectrum spread and random frequency hopping, CDMA can improve signal interference and access security over FDMA (frequency division multiple access) and TDMA (time division multiple access).

Two related key issues for CDMA channels are power control and interference reduction. Due to location diversity and mobile nature of users in satellite and wireless communications, signals arriving at the transmission site will have different power levels due to highly diversified signal fading due to deviations in traveling distances. Uneven powers cause imbalanced signal interferences, with some signals being overwhelmed by others and transmission interruptions. This scenario is further compounded by unnecessary power consumption due to competition from users to increase power levels to secure connection quality at the expense of others. As a result, to enhance network reliability and power consumption, it is essential that user transmission powers be actively controlled so that power consumption is minimized with a guaranteed level of signal interference.

The issues of interference suppression and power control have been actively pursued by many researchers. We refer the reader to [10∼13] for further discussions and references on this topic. In this paper, we study these issues on a platform of direct sequence CDMA (DS/CDMA). We show in this paper that at a high level of abstraction, the problem of interference suppression and power control can be formulated in a stochastic framework, as a minimization of total transmission power of multiple users under the constraint that overall signal interference among the users is maintained under a designated threshold.

Due to highly dynamic nature of user accessability of DS/CDMA in which users sign on and off frequently, and traveling distances between a user and the transmission channel entrance point change in real time (mobile users),

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the above optimization problem changes with time-varying uncertain system characterizations. Consequently, it must be solved efficiently in real time.

In this paper, we develop efficient recursive algorithms to solve the optimization in real time. Although the algorithms are described in a typical scenario of one base station with multiple users, they are scalable to deal with multiple station scenarios. A special case of such a process is a finite-state and homogeneous-in-time Markov chain that is irreducible and aperiodic. We assume that the channel gain is available, and ignore the quantization effect due to the limited bandwidth of the feedback channel. We also assume that the delay in the feedback channel is less than the time between two successive power control updates. As a result, interaction between two power control cycles is not present. In fact, more general bounded delays can be handled. However, the notation will become more complex. It appears to be more instructive that we confine ourselves to the aforementioned assumptions.

For effective solutions to the underlying optimization problem in real time, recursive algorithms of stochastic approximation type are developed to solve the constrained optimization problem. The signal interference is expressed as inequality constraints in the formulation. The inequality constraints are then converted to equality constraints by introducing slagging variables which are then treated by using ceratin Lagrange multipliers. Based on such a constrained optimization setup, and in view of the work [7], we develop stochastic approximation algorithms of penalty multiplier type. Convergence of the algorithms is established by weak convergence techniques developed recently in stochastic approximation methods [8]. In lieu of working with discrete iterates directly, appropriate continuous-time interpolation is employed. We show that the interpolated process converges weakly to a limit that can be characterized as the solution to an ordinary differential equation (ODE). The significance of the ordinary differential equation is that its stationary points are related to the Kuhn-Tucker points of the optimization problem.

One of the advantageous of this stochastic optimization approach is that the algorithm is on-line and can be implemented recursively. In addition, the channels need not be Markovian. Only stationary ergodicity is required. This ergodicity allows us to replace the instantaneous probability distribution by that of the invariant measure. Moreover, in contrast to the existing power control algorithms, we could allow the step size of recursive algorithms to be a constant (rather than a decaying sequence), which provides potential capability for tracking time-varying characteristics of the system.

The rest of the paper is arranged as follows. Section 2 presents the precise formulation of the CDMA system. Section 3 presents the proposed recursive algorithm. Section 4 is devoted to the study of asymptotic properties of the algorithm. It is shown that appropriately interpolated process converges to a limit that is a solution of an ordinary differential equation. A few more remarks are given in Section 5. Finally, an appendix is provided including the technical proofs.

Throughout the paper, we often use $K$ to denote a generic positive constant whose value may change for different appearances. For a vector or a matrix $v \in \mathbb{R}^{\ell\times r}$, $v'$ denotes its transpose. We also use $|\cdot|$ to denote either the norm of a vector or an absolute value. It should be clear from the context.

## 2 CDMA System Model

Consider a CDMA channel that is shared by $\kappa$ users simultaneously. For concreteness and notational simplicity, a basic binary DS/CDMA channel with additive white Gaussian noise (WGN) will be used to describe algorithms, although the methodology can be easily extended to more general DS/CDMA schemes. Assume that binary antipodal signals are used to transmit data from each user and transmission of data blocks of some pre-designed length $N$ is used for communications, known as spreading factor. The received baseband signal during one symbol interval is passed through a chip-matched filter followed by a chip-rate sampler.

We use index $k$ to represent user $k$. For each $k = 1,\dots,\kappa$, corresponding to the user $k$, and $i = 0,\dots$, representing the time index, the following symbols and assumptions are imposed:

- $\{s_k\}$: the normalized signature sequence, independent of time, satisfying $s_k \in \mathbb{R}^N$ satisfying $s'_k s_k = 1$;
- $p_k \in \mathbb{R}$ and $p_k \geq 0$: the transmitted power;
- $h_k \in \mathbb{R}$ and $h_k \geq 0$: the channel gain;
- $b_k(i) \in \mathbb{R}^N$: denotes the data bit of the $k$th user transmitted at time $i$, and $\{b_k(i)\}$ is a sequence of independent equiprobably random variables taking values $\pm 1$;
- $n(i) \in \mathbb{R}^N$: a standard $N$-dimensional Gaussian noise vector of zero mean and covariance $\sigma\sigma'$;
- $c_k \in \mathbb{R}^N$: a weighting vector;
- $r(i) \in \mathbb{R}^N$: the $i$th received symbol.

We refer the reader to [11] for further details on formulation for CDMA applications. At the $i$th received symbol, we have

$$r(i) = \sum_{k=1}^{\kappa} \sqrt{p_k h_k} b_k(i) + \sigma n(i). \tag{1}$$

The signal to interference ratio for the user $k$, which may be used as a quality of service, can be written as

$$\mathrm{SIR}_k = \frac{p_k h_k (c_k' s_k)^2}{\sum_{\substack{j=1\\ j\neq k}}^{\kappa} p_j h_j (c_k' s_j)^2 + c_k' \sigma\sigma' c_k}. \tag{2}$$

Note that in [10, 11], mean squares type criteria were used for code-aided interference suppression and adaptive implementations. In [13], mean squares algorithms were developed to minimize the error of $E|p(n)-p_*|^2$, where $p_*$ is the optimal power vector. Here we treat more general cost criteria with constraints. The constraints are handled through penalty multiplier methods.

If we followed the approach of [1, 18], we would use finite-state Markov chains to model the channels. Assume that there are $\kappa$ independent Markov chains, each of which represents one channel. Denote the state space of each Markov chain by $\mathcal{M}_c = \{1, \ldots, \ell\}$ for $k = 1, \ldots, \kappa$, where $\ell$ is a positive integer. Suppose that these Markov chains are time homogeneous with transition probability matrices given by $A_k$, $k = 1, \ldots, \kappa$, and that the channel gains are given by $h_{k,1}, \ldots, h_{k,\ell}$. These quantities may not be available, but their estimates $\widehat{A}_k$ and $\widehat{h}_{k,\iota}$ are readily obtainable. Assume that the Markov chains are irreducible and aperiodic. It then follows that corresponding to user $k$, there is a stationary distribution

$$\pi_k = (\pi_{k,1}, \ldots, \pi_{k,m_k}).$$

Consequently, there are $\ell^\kappa$ possible combinations of channels states. They can be thought of as a Markov chain with state space $\mathcal{M} = \mathcal{M}_c \times \cdots \times \mathcal{M}_c$, a $\kappa$-fold product of $\mathcal{M}_c$ with cardinality $m = |\mathcal{M}| = \ell^\kappa$. The ergodicity condition and the independence of different Markov chains mentioned above imply that there is a stationary distribution $\pi$ for the Markov chain with state space $\mathcal{M}$. This stationary distribution is of the form

$$\overline{\pi} = (\overline{\pi}_i) \stackrel{\text{def}}{=} \pi_1 \otimes \pi_2 \otimes \cdots \otimes \pi_\kappa,$$

where $a \otimes b$ denotes the Kronecker product [3].

Nevertheless, our approach to follow can handle more general channel models with non-Markovian structure. The only requirement is that the noise processes is averaged out in the sense of convergence in probability. In fact, the main ingredient of the Markovian setup is that there is an invariant measure. Without confining ourselves to the Markovian models, we can treat more general situation and with wider range of applications.

## 3 Constrained Optimization and Recursive Procedure

Suppose that the system under consideration has been in operation for an extended time. Thus, the ergodicity implies that we may replace the instantaneous probability distribution by that of the stationary probability. This stems from the idea of [13]. Use $\mathbb{1}$ to denote a column vector with all components being equal to 1. Denote the vector transmitted power by $p = (p_1, \ldots, p_\kappa)$. The power control problem can be formulated as a constrained optimization problem of the form:

$$\begin{aligned}
&\text{minimize } \ f(p) = \mathbb{1}' p = \sum_{k=1}^{\kappa} p_k,\\
&\text{subject to } \ g_k(p) \stackrel{\text{def}}{=} \gamma_k - \overline{\mathrm{SIR}}_k \le 0, \ \text{ for } k = 1, \ldots, \kappa,\\
&\overline{\mathrm{SIR}}_k = \int \mathrm{SIR}_k(x) d\mu(x),\\
&\mathrm{SIR}_k(x) = \frac{p_k h_k(x) (c_k' s_k)^2}{\sum_{\substack{j=1\\ j\neq k}}^{\kappa} p_j h_j(x) (c_k' s_j)^2 + c_k' \sigma\sigma' c_k},
\end{aligned} \tag{3}$$

where $\mu(\cdot)$ is the stationary invariant measure and $\gamma_k$ is the target SIR for user $k$.

For example, if the Markovian channels as mentioned in the previous section are considered, we have

$$\begin{aligned}
&\overline{\mathrm{SIR}}_k = \sum_{\iota=1}^{m} \mathrm{SIR}_{k,\iota} \overline{\pi}_\iota,\\
&\mathrm{SIR}_{k,\iota} = \frac{p_k h_{k,\iota} (c_k' s_k)^2}{\sum_{\substack{j=1\\ j\neq k}}^{\kappa} p_j h_{j,\iota} (c_k' s_j)^2 + c_k' \sigma\sigma' c_k}.
\end{aligned}$$

However, as mentioned, such a Markovian assumption is not needed in our formulation. Non-Markovian models can be treated.

In the above problem, the constraint appears in an inequality form. One way to solve the problem is to convert it to an equality constrained problem:

$$\begin{aligned}
&\text{minimize } \ f(p) = \mathbb{1}' p,\\
&\text{subject to } \ \varphi_k(w) = g_k(p) + z_k^2/2 = 0, \ \ k = 1, \ldots, \kappa,
\end{aligned} \tag{4}$$

where $z = (z_1, \ldots, z_\kappa)' \in \mathbb{R}^\kappa$ and $w = \begin{pmatrix} p \\ z \end{pmatrix} \in \mathbb{R}^{2\kappa \times 1}$.

In the actual application, when one wants to solve the constrained optimization problem (4), frequently, various quantities such as the transmitted power, the channel gains, and the normalized symbol sequence, cannot be measured or observed without error, and only noise corrupted observations or measurements are available. In this case, we have to use a stochastic optimization method to solve the problem. Following the approach in [7], we design the constrained stochastic optimization algorithm of penalty-multiplier type. For subsequent analyses, denote:

| | |
|---|---|
| $g_{k,p}(p)$: | the gradient of $g_k(\cdot)$ w.r.t. $p$ for user $k$ |
| $\varphi(w)$: | $(\varphi_1(w), \dots, \varphi_\kappa(w))'$ |
| $\Phi(p)$: | $(g_{1,p}(p), \dots, g_{\kappa,p}(p)) \in \mathbb{R}^{\kappa\times\kappa}$ |
| $\Psi(w)$: | $\sum_{k=1}^{\kappa} \varphi_k^2(w)/2$ |
| $Z(z)$: | $\operatorname{diag}(z_1, \dots, z_\kappa) \in \mathbb{R}^{\kappa\times\kappa}$ |
| $\widetilde{\Phi}(w)$: | $\begin{pmatrix}\Phi(p)\\ Z(z)\end{pmatrix} \in \mathbb{R}^{2\kappa\times\kappa}$ |
| $0_{\kappa\times 1}$: | a $\kappa$-dimensional 0 vector |
| $\widetilde{f}(p)$: | $\begin{pmatrix}f_p(p)\\ 0_{\kappa\times 1}\end{pmatrix} \in \mathbb{R}^{2\kappa\times 1}$ |
| $\mu$: | a fixed positive constant |
| $\eta(n)$ | $0 \le \eta(n) \to 0$ |
| $\chi(n)$: | $\chi(n) \in \mathbb{R}^{\kappa\times 1}$, $\chi_i(n) = \operatorname{sgn}(z_i)\vert_{z_i=z_i(n)}$ |
| $\lambda(n)$: | multiplier taking values in $\mathbb{R}^{\kappa\times 1}$ |
| $B^\dagger$: | Moore-Penrose psedoinverse of $B$. |

**Remark 3.1.**

(a) To construct a recursive algorithm for the minimization task, naturally, we need to use $f_p(p)$, the gradient of $f(p)$. However, in our case, $f_p(p(n))$ cannot be observed directly, but only noise corrupted gradient estimates $\widetilde{F}(n)$ are available. We assume the form $\widetilde{F}(n) = F(p(n), \varrho(n)) + \zeta(n)$. Note that for simplicity, we do not specify which gradient estimate methods are used, but only need the availability of $\widetilde{F}(n)$. Regardless of the methods of gradient estimates (such as likelihood, finite difference, random directions, infinitesimal perturbation methods etc.; see [8]), it is well known in stochastic approximation that the noise of gradient estimates of $f_p(p)$ will appear either in an additive form as $f_p(p)$+noise, or a nonadditive form as $F(p,$ noise) (where the nonadditive noise is averaged out to $f_p(p)$, i.e., $EF(p,$ noise$) = f_p(p)$). Our model is an extension of the above in that it includes both additive noise $\zeta(n)$ that can be unbounded and nonadditive noise $\varrho(n)$ that are bounded with $EF(p, \varrho(n)) = 0$ for each $p$. Our formulation is also more general than those that use least squares type criteria. For related multiuser detection problems in a least squares setup, we refer the reader to [10] and references therein.

(b) Denote $w(n) = \begin{pmatrix}p(n)\\ z(n)\end{pmatrix}$. Then, for example, the update for $p(n)$ can be computed as

$$p(n+1) = p(n) - \varepsilon[\widetilde{F}(n) + \Phi(p(n))\lambda(n) + \mu\Psi_p(w(n))], \tag{5}$$

where the last two terms in (5) are constructed following the penalty-multiplier method as suggested in [7].

(c) Note that in (5), we have implicitly assumed that each component of $p(n)$ is non-negative since it represents power. This can be easily done by using a projection scheme. If any of the components is ever less than 0, we reset it to a fixed point, say $0.5$, and restart the iterates. All subsequent developments still go through without modifications. More complex projections are possible, but for our purpose, the simple scheme will work. In what follows, for convenience we will not write the projection.

As usual in stochastic approximation, it is more convenient to center the observed quantities by their "mean values." Thus, we rewrite the observed values as their desired gradient term plus error terms. To construct sequences of estimates $w(n)$ and $\lambda(n)$, we write the algorithm as

$$\begin{cases} p(n+1) = p(n) - \varepsilon[f_p(p(n)) + \xi(n) + \zeta(n) \\ \qquad\qquad + \Phi(p(n))\lambda(n) + \mu\Psi_p(w(n))] \\ z(n+1) = z(n) - \varepsilon[Z(z(n))\lambda(n) + \mu\Psi_z(w(n))] \\ \qquad\qquad + \varepsilon\eta(n)\chi(n) \\ \lambda(n) = -\widetilde{\Phi}^\dagger(w(n))[\widetilde{f}_p(p(n)) + \widetilde{\xi}(n) + \widetilde{\zeta}(n)] \\ \xi(n) = F(p(n), \varrho(n)) - f_p(p(n)), \end{cases} \tag{6}$$

where $\widetilde{\Phi}^\dagger(w)$ denotes the psedoinverse of $\widetilde{\Phi}(w)$, $\widetilde{\xi} = \begin{pmatrix}\xi\\ 0\end{pmatrix} \in \mathbb{R}^{2\kappa\times 1}$, and $\widetilde{\zeta} = \begin{pmatrix}\zeta\\ 0\end{pmatrix} \in \mathbb{R}^{2\kappa\times 1}$. Note that $\lambda(n)$ is chosen so that

$$\left|\widetilde{f}_p(p(n)) + \widetilde{\xi}(n) + \widetilde{\zeta}(n) + \widetilde{\Phi}^\dagger(w(n))\lambda\right|^2 \tag{7}$$

is minimized.

As explained in [7], the term $\varepsilon\eta(n)\chi(n)$ plays the role of destabilizing the algorithm near points $p$, where there is some $\lambda$ whose components are not all nonnegative such that $f_p(p) + \Phi(p)\lambda = 0$. Its purpose is to eliminate the non Kuhn-Tucker points that the algorithm might converges to. The criterion for the selection of $\lambda(n)$ is close to some $\overline{\lambda}$ for which $\widetilde{f}_p(p) + \widetilde{\Phi}(w)\overline{\lambda} = 0$ is "nearly" satisfied.

In lieu of fixed step size algorithm, we may also consider decreasing step size algorithm of the form:

$$\begin{cases} p(n+1) = p(n) - a(n)(f_p(p(n)) + \xi(n) + \zeta(n) \\ \qquad\qquad + \Phi(p(n))\lambda(n) + \mu\Psi_p(w(n))) \\ z(n+1) = z(n) - a(n)(Z(z(n))\lambda(n) + \mu\Psi_z(w(n))] \\ \qquad\qquad + a(n)\eta(n)\chi(n)) \\ \lambda(n) = -\widetilde{\Phi}^{\dagger}(w(n))\widetilde{f}_p(p(n)) \\ \xi(n) = F(p(n), \varrho(n)) - f_p(p(n)), \end{cases} \tag{8}$$

where $\{a(n)\}$ is a sequence of positive real numbers satisfying $a(n) \to 0$ as $n \to \infty$ and $\sum_n a(n) = \infty$.

The precise conditions needed will be given later. Compared with the formulation in [7], in addition to additive noise, nonadditive noise sequences are also considered in our paper to capture the features arising in the wireless communication applications. Our task to follow is to obtain the convergence of the algorithm. Different from the approach in [7], martingale averaging approach will be used in the asymptotic study.

## 4 Asymptotic Properties of the Recursive Algorithm

This section is devoted to the asymptotic properties of the constrained stochastic optimization algorithm. For simplicity, we take $w(0) = w_0 = (p^0, z^0)$ that is nonrandom.

(A1) $\{\varrho(n)\}$ is a bounded $\phi$-mixing sequence with mixing rate $\check{\phi}(n)$ satisfying $\sum_k \check{\phi}^{1/2}(k) < \infty$. $\{\zeta(n)\}$ is a martingale difference sequence satisfying $E|\zeta(n)|^2 < \infty$.

(A2) $F(\cdot, \varrho)$ is continuous for each $\varrho$.

(A3) $\Phi(p)\varphi(w) = 0$ implies that $\varphi(w) = 0$. Define the set $\widehat{G} = \{p$ : there is a $\lambda$ satisfying $\mathbb{1} + \Phi(p)\lambda = 0$ where $\lambda_k g_k(p) = 0$ for $k = 1, \dots, \kappa\}$. $\widehat{G}$ is bounded and closure of its interior.

**Remark 4.1.** Note that in view of the formulation in (4) and the definition of $g_k(\cdot)$ given in (3), $g_k(\cdot)$ is at least twice continuously differentiable.

Recall that (see [6, p.115]) a constraint $g_k(\cdot)$ is said to be active at $p$ if $g_k(p) = 0$. Let $A(p)$ be the set of indices active constraints at $p \in \partial\widehat{G}$. A point $p \in G$ is said to be a Kuhn-Tucker point if there exist $\lambda_k \geq 0$ such that

$$f_p(p) + \sum_{k \in A(p)} \lambda_k g_{k,p}(p) = 0.$$

If at $p$ the constraint is inactive, i.e., $g_k(p) < 0$, $\lambda_k = 0$. Define $\widehat{G}^+$ to be the set of points in $\widehat{G}$ that are Kuhn-Tucker points.

Now consider the recursive algorithm. Define the interpolated sequences as

$$p^{\varepsilon}(t) = p(n),\ z^{\varepsilon}(t) = z(n),\ w^{\varepsilon}(t) = w(n), \tag{9}$$

for $t \in [n\varepsilon, n\varepsilon + \varepsilon)$. We obtain the following theorem, whose proof is in the appendix.

**Theorem 4.2.** Assume (A1)–(A2). Then $\{w^{\varepsilon}(\cdot)\} = \{p^{\varepsilon}(\cdot), z^{\varepsilon}(\cdot)\}$ converges weakly to $w^{\varepsilon}(\cdot) = (p(\cdot), z(\cdot))$ such that it is the solution of the system ordinary differential equations

$$\begin{aligned} &\dot{p}(t) = -\mathbb{1} - \Phi(p(t))\overline{\lambda}(t),\ p(0) = p^0, \\ &\dot{z}(t) = -Z(z(t))\overline{\lambda}(t),\ z(0) = z^0 \\ &\overline{\lambda}(t) = -[\widetilde{\Phi}'(p(t))\widetilde{\Phi}(p(t))]^{-1}\widetilde{\Phi}'(p(t)) \begin{pmatrix} \mathbb{1} \\ 0 \end{pmatrix}. \end{aligned} \tag{10}$$

Moreover, the stationary points of (10) are in $\widehat{G}^+$ Furthermore, if the set of stationary points is a singleton set $\{p^*\}$, then $p^{\varepsilon}(\cdot) \to p^*$ in probability.

**Remark 4.3.** If we use the notation

$$G = \{p : g_k(p) \leq 0,\ k = 1, \dots, \kappa\},$$

the algorithm may be written by use of a projection operator. Define $\prod_G$ to be the projection onto $G$. That is, $\prod_G(y)$ is the closest point in $G$ to $y$. The algorithm than can be written in terms of this projection operator; see [8].

Likewise, we can also derive the following result. The piecewise constant interpolation is modified for the interpolation interval $[t_n, t_{n+1})$ defined as follows. Let $t_n = \sum_{kk=0}^{n-1} \varepsilon_k$, and define $m(t) = \max\{n; t_n \leq t\}$. Define

$$\begin{aligned} &p^0(t) = p(n),\ z^0(t) = z(n),\ w^0(t) = w(n), \\ &p^n(t) = p^0(t_n + t),\ z^n(t) = z^0(t_n + t), \\ &w^n(t) = w^0(t_n + t),\ \ t \in [t_n, t_{n+1}). \end{aligned}$$

In lieu of $w^{\varepsilon}(\cdot)$, we work with $w^n(\cdot)$ and derive the following convergence result. The proof of the result follows the same kind of argument as that of Theorem 4.2 with modifications of the notation mentioned above and is thus omitted.

**Theorem 4.4.** Assume that the conditions of Theorem 4.2 are satisfied. In lieu of (6), consider (8). The conclusion of Theorem 4.2 continue to hold.

## 5 Conclusion

This paper has focused on power control problems via a stochastic optimization approach. Constrained stochastic optimization algorithms has been proposed and analyzed. Asymptotic properties of the algorithms have been obtained.

The algorithms studied aim to obtaining the local constrained minimizer. Related problems concerning constrained global optimization can be considered. Nevertheless, it is known that the global optimization algorithms have slower convergence rates compared with the local algorithm counter parts (see [14, 15]). For future study, accessing convergence rates taking into consideration of computational budget, and balancing the bias and noise effect (see [9]) are worthy of further investigation. To improve the efficiency of the algorithms is another important issue that needs to be addressed. By and large, the choice of step size sequences is usually not a simple matter. To ease the difficulty, iterate averaging methods may be used (see [16] and see also [17]) via an averaging approach in the sense suggested by Polyak and Ruppert (see [8, Chapter 11]). All of these require further study and investigation.

## A Appendix

This section provides the detailed proof of the convergence theorem. Since Theorem 4.4 can be proved similar to that of Theorem 4.2 via the modification of the definitions of interpolations, we shall only give the verbatim argument for the proof of Theorem 4.2. In fact, our main effort will be directed to deriving the limit ordinary differential equations.

**Proof of Theorem** 4.2**.** The proof is divided into several steps. In fact, once the limit ODE is obtained, we can use the result in [6] to complete the proof. To carry out the analysis, we take a $\nu$-truncation (see [8, p. 282]) and consider a truncated random process in lieu of the original one. That is, for arbitrary $\nu > 0$, denote by $S_\nu = \{x : |x| \le \nu\}$ the sphere with radius $\nu$, and let $w^{\varepsilon,\nu}(\cdot) = (p^{\varepsilon,\nu}(\cdot), z^{\varepsilon,\nu}(\cdot))$ be the process that is equal to $w^\varepsilon(\cdot)$ up until the first exit from $S_\nu$. Then we work with $\{w^{\varepsilon,\nu}(\cdot)\}$, and derive its tightness and weak convergence. Finally, we let $\nu \to \infty$ to conclude the proof.

Before proceeding further, let us point out several properties satisfied by the functions involved in the stochastic optimization problem.

i) In view of the defining relation (3), for each $k = 1, \dots, \kappa$, $g_k(\cdot)$ has bounded and continuous second partial derivatives.

ii) The set $\{g_{k,p}(p)$ such that $g_k(p) = 0\}$ are linearly independent for each $p$.

iii) $f_p(p) = \mathbb{1}$.

iv) Arguing along the line of [7, p.201], we can show that by use of (7), $\widetilde{\Phi}'(w)\widetilde{\Phi}(w)$ is nonsingular, and hence, we may choose

$$\lambda(n) = -[\widetilde{\Phi}'(w^\nu(n))\Phi(w^\nu(n))]^{-1}\widetilde{\Phi}'(w^\nu(n)) \times(\widetilde{f}(w^\nu(n)) + \widetilde{\xi}_n + \widetilde{\zeta}(n)).$$

Step 1 (Tightness). Using $w = \begin{pmatrix} p \\ z \end{pmatrix}$ and the truncation, rewrite (6) as

$$\begin{aligned} w^\nu(n+1) &= w^\nu(n) - \varepsilon\Big[\widetilde{f}_p(p^\nu(n)) + \widetilde{\xi}(n) + \widetilde{\zeta}(n) \\ &\quad +\mu\Psi_w(w^\nu(n)) + \begin{pmatrix} \Phi(p(n))\lambda(n) \\ Z(z^\nu(n))\lambda(n) \end{pmatrix}\Big] q_\nu(w^\nu(n)) \\ &\quad +\varepsilon\eta(n)\begin{pmatrix} 0 \\ \chi(n) \end{pmatrix} q_\nu(w^\nu(n)), \end{aligned} \tag{a1}$$

where $q_\nu(w) = 1$ for $w \in S_\nu$ and $q_\nu(w) = 0$ for $w \in \mathbb{R}^{2\kappa\times 1} - S_{\nu+1}$.

For any $\delta > 0$ and $0 \le s \le \delta$, use $E_t^\varepsilon$ to denote the conditional expectation with respect to the $\sigma$-algebra $\mathcal{F}_t^\varepsilon$ generated by $\{w(0), \varrho(j), \zeta(j), j \le t/\varepsilon\}$. We have that

$$\begin{aligned} &E_t^\varepsilon\,|w^{\varepsilon,\nu}(t+s) - w^{\varepsilon,\nu}(t)|^2 \\ &\le K\varepsilon E_t^\varepsilon\Big|\sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1}[\widetilde{f}_p(p^\nu(n)) + \widetilde{\xi}(k) + \widetilde{\zeta}(k) \\ &\quad +\mu\Psi_w(w^\nu(k))]q_\nu(w^\nu(k))\Big|^2 \\ &\quad +K\varepsilon E_t^\varepsilon\Big|\sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1}\begin{pmatrix} \Phi(p(n))\lambda(n) \\ Z(z^\nu(n))\lambda(n) \end{pmatrix} q_\nu(w^\nu(k))\Big|^2 \\ &\quad +K\varepsilon E_t^\varepsilon\Big|\sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1}\eta(k)\begin{pmatrix} 0 \\ \chi(k)q_\nu(w^\nu(k)) \end{pmatrix}\Big|^2. \end{aligned}$$

By the boundedness of $w^\nu(k)$ and the continuity of $\widetilde{f}_p(\cdot)$,

$$E_t^\varepsilon\Big|\varepsilon\sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1}\widetilde{f}_p(p^\nu(k))\Big|^2 \le K\varepsilon^2\Big(\frac{t+s}{\varepsilon} - \frac{t}{\varepsilon}\Big)^2 \le K\delta^2.$$

Similarly,

$$E_t^\varepsilon \left| \varepsilon \sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1} \Psi_w(w^\nu(k)) q_\nu(w^\nu(k)) \right|^2 \le K\delta^2,$$

$$\varepsilon E_t^\varepsilon \left| \sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1} \begin{pmatrix} \Phi(p^\nu(n))\lambda(n) \\ Z(z^\nu(n))\lambda(n) \end{pmatrix} q_\nu(w^\nu(k)) \right|^2 \le K\delta^2,$$

$$E_t^\varepsilon \left| \varepsilon \sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1} \eta(k)\chi(k) q_\nu(w^\nu(k)) \right|^2 \le K\eta(t/\varepsilon)\delta^2.$$

Recall the definition of $\xi(n)$ and by the boundedness of $\{\varrho(n)\}$ and the continuity of $F(\cdot,\varrho)$ for each $\varrho$, we also have

$$E_t^\varepsilon \left| \varepsilon \sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1} \widetilde{\xi}(k) q_\nu(w^\nu(k)) \right|^2 \le K\delta^2.$$

Since $\{\widetilde{\zeta}(n)\}$ is a sequence of martingale difference sequence with finite second moment,

$$E_t^\varepsilon \left| \varepsilon \sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1} \widetilde{\zeta}(k) \right|^2 = E_t^\varepsilon \varepsilon^2 \sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1} |\widetilde{\zeta}(k)|^2 = O(\varepsilon)O(\delta^2).$$

Putting the above estimates together, we arrive at

$$\lim_{\delta\to 0} \limsup_{\varepsilon\to 0} E|w^{\varepsilon,\nu}(t+s) - w^{\varepsilon,\nu}(t)|^2 = 0. \tag{a2}$$

By the tightness criterion (see [6, Theorem 3, p.47], or [8, Chapter 7]), $\{w^{\varepsilon,\nu}(\cdot)\}$ is tight. By the Cramer-Wold device, $\{p^{\varepsilon,\nu}(\cdot)\}$ and $\{z^{\varepsilon,\nu}(\cdot)\}$ are both tight.

Step 2. (Characterization of limit: Averaging). Since $\{w^{\varepsilon,\nu}(\cdot)\} = \{p^{\varepsilon,\nu}(\cdot), z^{\varepsilon,\nu}(\cdot)\}$ is tight, by Prohorov's theorem, we can extract convergent subsequences. Select such a subsequence and still denote it by $(p^{\varepsilon,\nu}(\cdot), z^{\varepsilon,\nu}(\cdot))$ without loss of generality. Denote the limit by $(p^\nu(\cdot), z^\nu(\cdot))$. We shall characterize this limit process. By virtue of the Skorohod representation, we may assume that the $(p^{\varepsilon,\nu}(\cdot), z^{\varepsilon,\nu}(\cdot)) \to (p^\nu(\cdot), z^\nu(\cdot))$ w.p.1, and the convergence is uniform on each bounded time interval. Choose a subsequence $\{m_\varepsilon\}$ such that $m_\varepsilon \to \infty$, but $\varepsilon m_\varepsilon \to 0$ as $\varepsilon \to 0$. Using this sequence to subdivide the interval $[t/\varepsilon, (t+s)/\varepsilon)$ into subintervals. Note that here and hereafter, $t/\varepsilon$, $(t+s)/\varepsilon$ are assumed to be integers. This is no loss in generality since we can always take their integer part any way.

To complete the proof, we shall show that $M^\nu(\cdot)$ defined

$$M^\nu(t) = w^\nu(t) - w^\nu(0) + \int_0^t \left[ \widetilde{f}_p(p^\nu(u)) + \begin{pmatrix} \Phi(p^\nu(u))\overline{\lambda}(u) \\ Z(z^\nu(u))\lambda(u) \end{pmatrix} \right] q_\nu(w^\nu(u)) du \tag{a3}$$

is a continuous-time martingale with Lipschitz continuous sample paths w.p.1. Then, by virtue of [8, Theorem 4.1.1, p.98], $M^\nu(\cdot)$ must be a constant w.p.1. However, $M^\nu(0) = 0$. Thus, $M^\nu(t) \equiv 0$, and as a result,

$$w^\nu(t) = w^\nu(0) - \int_0^t \left[ \widetilde{f}_p(p^\nu(u)) + \begin{pmatrix} \Phi(p^\nu(u))\overline{\lambda}(u) \\ Z(z^\nu(u))\overline{\lambda}(u) \end{pmatrix} \right] \times q_\nu(w^\nu(u)) du.$$

Thus, we need only verify the martingale property of $M^\nu(\cdot)$.

To verify the martingale property, we shall show that for any bounded and continuous function $h(\cdot)$, positive integer $m_0$, and $0 < t_i \le t \le t+s$ with $i \le m_0$,

$$\begin{aligned} &E\prod_{i=1}^{m_0} h(w^\nu(t_i))[M^\nu(t+s) - M^\nu(t)] \\ &= -E\prod_{i=1}^{m_0} h(w^\nu(t_i)) \int_t^{t+s} \left[ \widetilde{f}_p(p^\nu(u)) + \begin{pmatrix} \Phi(p^\nu(u))\overline{\lambda}(u) \\ Z(z^\nu(u))\lambda(u) \end{pmatrix} \right] q_\nu(w^\nu(u)) du. \end{aligned} \tag{a4}$$

To prove (a4), we begin with the process $w^{\varepsilon,\nu}(\cdot)$ By virtue of the weak convergence and the Skorohod representation,

$$\begin{aligned} &\lim_{\varepsilon\to 0} E\prod_{i=1}^{m_0} h(w^{\varepsilon,\nu}(t_i))[M^{\varepsilon,\nu}(t+s) - M^{\varepsilon,\nu}(t)] \\ &= E\prod_{i=1}^{m_0} h(w^\nu(t_i))[M^\nu(t+s) - M^\nu(t)]. \end{aligned} \tag{a5}$$

Since $f_p(p) = \mathbb{1}$, we have

$$\begin{aligned} &\lim_{\varepsilon\to 0} E\prod_{i=1}^{m_0} h(w^{\varepsilon,\nu}(t_i)) \left[ \sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1} \varepsilon \widetilde{f}_p(p^{\varepsilon,\nu}(k)) q_\nu(w^\nu(k)) \right] \\ &= E\prod_{i=1}^{m_0} h(w^\nu(t_i)) \mathbb{1} \int_t^{t+s} q_\nu(w^\nu(u)) du. \end{aligned} \tag{a6}$$

Next,

$$\varepsilon \sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1} \xi(k)q_\nu(w^\nu(k))$$
$$= \sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1} \Delta_\varepsilon \frac{1}{\Delta_\varepsilon} \sum_{k=im_\varepsilon}^{im_\varepsilon+m_\varepsilon-1} F(p^{\varepsilon,\nu}(k),\varrho(k))q_\nu(w^\nu(k))$$
$$= \sum_{im_\varepsilon=t/\varepsilon}^{(t+s)/\varepsilon-1} \Delta_\varepsilon \frac{1}{\Delta_\varepsilon} \sum_{k=im_\varepsilon}^{im_\varepsilon+m_\varepsilon-1} F(p^{\varepsilon,\nu}(im_\varepsilon),\varrho(k))$$
$$\times q_\nu(w^\nu(k)) + o(1), \tag{a7}$$

where $o(1) \to 0$ in probability uniformly in $t$ due to the continuity of $F(\cdot,\varrho)$ for each $\varrho$. Thus using the mixing property of the noise, we obtain

$$\lim_{\varepsilon\to 0} E\prod_{i=1}^{m_0} h(w^{\varepsilon,\nu}(t_i)) \sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1} \varepsilon \begin{pmatrix}\xi(k)\\ 0\end{pmatrix}$$
$$= \lim_{\varepsilon\to 0} E\prod_{i=1}^{m_0} h(w^{\varepsilon,\nu}(t_i)) \sum_{im_\varepsilon=t/\varepsilon}^{(t+s)/\varepsilon-1} \Delta_\varepsilon$$
$$\times \frac{1}{\Delta_\varepsilon} \sum_{k=im_\varepsilon}^{im_\varepsilon+m_\varepsilon-1} E_{im_\varepsilon} \begin{pmatrix} F(p^{\varepsilon,\nu}(im_\varepsilon),\varrho(k)) \\ 0 \end{pmatrix} q_\nu(w^\nu(k))$$
$$= E\prod_{i=1}^{m_0} h(w^\nu(t_i)) \int_t^{t+s} \begin{pmatrix}\overline{F}(p^\nu(u))\\ 0\end{pmatrix} q_\nu(w^\nu(u))du. \tag{a8}$$

In deriving the limit above, there are some delicate details. For a reader who is interested in this, see [8]. As for the martingale difference sequence, it is plain that

$$\lim_{\varepsilon\to 0} E\prod_{i=1}^{m_0} h(w^{\varepsilon,\nu}(t_i)) \left[\sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1} \zeta(k)\right] = 0. \tag{a9}$$

It can shown as in [7, p.201],

$$\lambda^\nu(n) = -[\widetilde{\Phi}'(p^\nu(n))\widetilde{\Phi}(p^\nu(n))]^{-1}\widetilde{\Phi}'(p^\nu(n))$$
$$\times(\widetilde{f}_p(p^\nu(n)) + \widetilde{\xi}(n) + \widetilde{\zeta}(n)). \tag{a10}$$

For convenience, denote

$$\Lambda(p^\nu(n)) = -\widetilde{\Phi}(p^\nu(n))[\widetilde{\Phi}'(p^\nu(n))\widetilde{\Phi}(p^\nu(n))]^{-1}$$
$$\times\widetilde{\Phi}'(p^\nu(n)).$$

We claim that

$$\lim_{\varepsilon\to 0} E\prod_{i=1}^{m_0} h(w^{\varepsilon,\nu}(t_i)) \Big[\varepsilon \sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1} \Lambda(p^\nu(k))(\widetilde{f}_p(p^\nu(k))$$
$$+\widetilde{\xi}(k) + \widetilde{\zeta}(k))q_\nu(w^\nu(k))\Big]$$
$$= E\prod_{i=1}^{m_0} h(w^\nu(t_i)) \int_t^{t+s} \widetilde{\Phi}(w^\nu(u))\overline{\lambda}^\nu(w^\nu(u))$$
$$\times q_\nu(w^\nu(u))du, \tag{a11}$$

where $\overline{\lambda}(w^\nu(u))$ is given in (10).

We examine each of the terms in (a11) in detail. First, note that for each $p$, $f_p(p) = \mathbb{1}$ and so $\widetilde{f}_p(p) = \begin{pmatrix}\mathbb{1}\\ 0\end{pmatrix}$. Similar to the derivation of the limit in (a6), we can prove

$$\lim_{\varepsilon\to 0} E\prod_{i=1}^{m_0} h(w^{\varepsilon,\nu}(t_i)) \sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1} \varepsilon\Lambda(p^\nu(k))\widetilde{f}_p(p^\nu(k))$$
$$= E\prod_{i=1}^{m_0} h(w^\nu(t_i)) \int_t^{t+s} \widetilde{\Phi}(w^\nu(u))\overline{\lambda}(w^\nu(u))q_\nu(w^\nu(u))du.$$

As for the martingale difference noise term, we have

$$\lim_{\varepsilon\to 0} E\prod_{i=1}^{m_0} h(w^{\varepsilon,\nu}(t_i)) \left[\sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1} \varepsilon\Lambda(p^\nu(k))\widetilde{\zeta}(k)\right]$$
$$= \lim_{\varepsilon\to 0} E\prod_{i=1}^{m_0} h(w^{\varepsilon,\nu}(t_i)) \Big[\sum_{im_\varepsilon=t/\varepsilon}^{(t+s)/\varepsilon-1} \Delta_\varepsilon$$
$$\times \frac{1}{\Delta_\varepsilon}\sum_{k=im_\varepsilon}^{im_\varepsilon+m_\varepsilon-1} \Lambda(p^\nu(k))\widetilde{\zeta}(k)\Big]$$
$$= \lim_{\varepsilon\to 0} E\prod_{i=1}^{m_0} h(w^{\varepsilon,\nu}(t_i)) \Big[\sum_{im_\varepsilon=t/\varepsilon}^{(t+s)/\varepsilon-1} \Delta_\varepsilon \Lambda(p^\nu(im_\varepsilon))$$
$$\times\Big[\frac{1}{\Delta_\varepsilon}\sum_{k=im_\varepsilon}^{im_\varepsilon+m_\varepsilon-1} \widetilde{\zeta}(k)\Big]\Big] = 0.$$

For the terms involving the nonadditive noise $\widetilde{\xi}(n)$, we again use the idea of "separation" to obtain

$$\lim_{\varepsilon\to0} E\prod_{i=1}^{m_0} h(w^{\varepsilon,\nu}(t_i))\Big[\sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1}\varepsilon\Lambda(p^\nu(k))\times\begin{pmatrix}F(p^\nu(k),\varrho(k))\\0\end{pmatrix}\Big]$$

$$\lim_{\varepsilon\to0} E\prod_{i=1}^{m_0} h(w^{\varepsilon,\nu}(t_i))\Big[\sum_{k=im_\varepsilon}^{im_\varepsilon+m_\varepsilon-1}\Delta_\varepsilon\Lambda(p^\nu(im_\varepsilon))\frac{1}{\Delta_\varepsilon}\times\begin{pmatrix}E_{im_\varepsilon}F(p^\nu(im_\varepsilon),\varrho(k))\\0\end{pmatrix}\Big]=0.$$

Note that in the above, we have used that for each fixed $x$,

$$\frac{1}{\Delta_\varepsilon}\sum_{k=im_\varepsilon}^{im_\varepsilon+m_\varepsilon-1}\begin{pmatrix}E_{im_\varepsilon}F(x,\varrho(k))\\0\end{pmatrix}\to0 \text{ in probability,}$$

and the dominated convergence theorem.

Step 3. (Estimate of an energy function). Next, we claim that

$$\lim_{\varepsilon\to0} E\prod_{i=1}^{m_0} h(w^{\varepsilon,\nu}(t_i))\left[\varepsilon\sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1}\mu\Psi_w(w^\nu(k))\right]=0. \tag{a12}$$

Note that $\Psi(w)$ can be considered as an energy function. Moreover,

$$\Psi_w(w)=\varphi'(w)\widetilde{\Phi}'_w(w).$$

Direct calculation reveals that

$$\begin{aligned}\Psi(w^\nu(n+1))-\Psi(w^\nu(n))\le&-\varepsilon\varphi'(w^\nu(n))\widetilde{\Phi}'_w(w^\nu(n))\\&\Big[(\mathbb{1}+\widetilde{\xi}(n)+\widetilde{\zeta}(n))+\mu\Psi_w(w^\nu(n))\\&+\widetilde{\Phi}(w^\nu(n))\lambda(n)\Big]\\&+O(\varepsilon^2)[|\widetilde{\xi}(n)|^2+|\widetilde{\zeta}(n)|^2+1].\end{aligned}$$

Note that for some $k_0>0$,

$$|\Psi_w(w^\nu(n))|^2\le k_0\Psi(w^\nu(n)),$$

and

$$\begin{aligned}&\varphi'(w^\nu(n))\widetilde{\Phi}'(w^\nu(n))\widetilde{\Phi}(w^\nu(n))\lambda(n)\\&=\varphi'(w^\nu(n))[\widetilde{\Phi}'(w^\nu(n))\widetilde{\Phi}(w^\nu(n))]\\&\quad\times[\widetilde{\Phi}'(w^\nu(n))\widetilde{\Phi}(w^\nu(n))]^{-1}\widetilde{\Phi}'(w^\nu(n))(\mathbb{1}+\widetilde{\xi}(n)+\widetilde{\zeta}(n)).\end{aligned}$$

Upon cancellation, we obtain

$$\begin{aligned}\Psi(w^\nu(n+1))-\Psi(w^\nu(n))\le&\ \varepsilon k_0\mu\Psi(w^\nu(n))\\&+O(\varepsilon^2)[|\widetilde{\xi}(n)|^2+|\widetilde{\zeta}(n)|^2+1].\end{aligned} \tag{a13}$$

For sufficiently small $\varepsilon>0$, we can make $0<\varepsilon k_0\mu<1$. Taking expectation in (a13) and using the boundedness of $\{\varrho(n)\}$ and the continuity of $F(\cdot,\varrho)$, we have that

$$E\Psi(w^\nu(n+1))\le(1-k_0\mu\varepsilon)E\Psi(w^\nu(n))+O(\varepsilon^2).$$

Iterating of the inequality above leads to

$$E\Psi(w^\nu(n+1))=(1-k_0\mu\varepsilon)^nE\Psi(w^\nu(0))+O(\varepsilon), \tag{a14}$$

and hence $E\Psi(w^\nu(n+1))=O(\varepsilon)$. Using (a14) together with dominated convergence theorem, we have

$$\begin{aligned}&E\prod_{i=1}^{m_0} h(w^{\varepsilon,\nu}(t_i))\left[\sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1}\varepsilon\Psi_w(w^\nu(k))q_\nu(w^\nu(k))\right]\\&=E\prod_{i=1}^{m_0} h(w^{\varepsilon,\nu}(t_i))\frac{\partial}{\partial w}\left[\sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1}\varepsilon\Psi_w(w^\nu(k))q_\nu(w^\nu(k))\right]\\&-E\prod_{i=1}^{m_0} h(w^{\varepsilon,\nu}(t_i))\left[\sum_{k=t/\varepsilon}^{(t+s)/\varepsilon-1}\varepsilon\Psi(w^\nu(k))\frac{\partial}{\partial w}q_\nu(w^\nu(k))\right]\\&\to0 \text{ as } \varepsilon\to0.\end{aligned}$$

Thus, (a12) is proven.

Step 4. (Characterization of limit (completion). Combining Step 1, Step 2, and Step 3, we arrive at that $w^{\varepsilon,\nu}(\cdot)$ converges weakly to $w^\nu(\cdot)(p^\nu(\cdot),z^\nu(\cdot))$ such that $p^\nu(\cdot)$ and $z^\nu(\cdot)$ are the solutions of the following differential equations.

$$\begin{aligned}&\dot{p}^\nu(t)=-\mathbb{1}-\Phi(p^\nu(t))\overline{\lambda}^\nu(t)q_\nu(w^\nu(t)),\ p(0)=p^0,\\&\dot{z}^\nu(t)=-Z(z^\nu(t))\overline{\lambda}^\nu(t)q_\nu(w^\nu(t)),\ z(0)=z^0\\&\overline{\lambda}^\nu(t)=-[\widetilde{\Phi}'(p^\nu(t))\widetilde{\Phi}(p^\nu(t))]^{-1}\widetilde{\Phi}'(p^\nu(t))\begin{pmatrix}\mathbb{1}\\0\end{pmatrix}\\&\qquad\times q_\nu(w^\nu(t)).\end{aligned} \tag{a15}$$

Step 5. (Passing the limit as $\nu\to\infty$). Let $P^0(\cdot)$ and $P^\nu(\cdot)$ be the measures induced by $w(\cdot)$ and $w^\nu(\cdot)$, respectively. Since the differential equation in (10) has a unique solution for each initial condition, $P^0(\cdot)$ is unique. For each $T<\infty$ and $t\le T$, $P^0(\cdot)$ agrees with $P^\nu(\cdot)$ on all Borel subsets of the set of paths in $D^{2\kappa}[0,\infty)$ with values in $S_\nu$. By using $P^0(\sup_{t\le T}|w(t)|\le\nu)\to1$ as $\nu\to\infty$, and the weak convergence of $w^{\varepsilon,\nu}(\cdot)$ to $w^\nu(\cdot)$, $w^\varepsilon(\cdot)$ converges weakly to $w(\cdot)$. The argument is similar to that of [8, pp. 282-285] and the details are omitted.

Step 6. (Characterization of Kuhn-Tucker points.) This part follows the same argument as in [6, p.115]. In addition, the result about the singleton set also follows. □

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