

# Delay-Constrained Energy-Efficient Wireless Packet Scheduling with QoS Guarantees

Xiliang Zhong and Cheng-Zhong Xu

Department of Electrical and Computer Engineering

Wayne State University, Detroit, Michigan 48202

{xlzhong, czxu}@wayne.edu

**Abstract**—This paper considers packet scheduling with delay constraint that minimizes the average transmission energy expenditure in a wireless environment. There were recent studies that modeled transmission rate as a linear combination of input packet arrivals in the derivation of optimal scheduling policies for independent input arrivals. In this paper, we extend the scheduling policies in two aspects. We first consider the impact of input time-correlation and derive an optimal scheduling policy for energy minimization. It is observed that existing energy optimized scheduling policies save energy by transmitting packets slowly. They may result in unexpected deadline misses. We establish a relationship between the maximum reliable transmission rate and a QoS constraint, characterized by a deadline miss rate. The relationship facilitates the trade-off study of power consumption and delay constraint. Simulation results verify the relationship and show the superiority of the scheduler in comparison with the existing approaches.

## I. INTRODUCTION

Ubiquitous communication using wireless devices is becoming a reality. The current and future wireless system will support a variety of data, such as voice, video, web, etc. Packet-switched wireless communication techniques have developed significantly in the past years for efficient data transmission, most notably in wireless LANs, ad hoc and sensor networks. Most of the wireless devices are powered by limited battery resources; reliable content delivery over a wireless channel is a major source of energy expenditure. In this paper, we study the tradeoff between transmission power and delay constraints.

It has been observed that power consumption between two points over a wireless channel is exponentially related to the information transmission rate [2], [7], [10]. By transmitting over a longer period of time at a lower transmission rate, the average energy required for reliable transmission of a packet can be significantly reduced. However, applications are usually delay-sensitive. The transmission time can not be arbitrarily long. There have been different approaches to dealing with the energy delay tradeoff. In [3], [2], [10], average queuing delay was considered as a constraint in energy minimization. In [12], [6], a single deadline for all packets was set so that all arrivals in time  $[0, T)$  had to be transmitted before time  $T$ . Both average delay and deadline constraints were considered in [8] with a focus on the impact of battery energy recovery. More recently, explicit delay constraint to each packet was used in which packet transmission must finish before its

deadline [7]. The scheduling process was modeled with an elegant linear structure. The relation enables the construction of robust schedulers and derivation of the first bounds on power consumption. The results form an important step towards understanding of energy-efficient packet scheduling for delay-sensitive applications.

In this paper, we put delay constraint to each packet as in [7]. The main contributions are two-fold. First, we consider the impact of input correlation structure on scheduling policy. We present an optimal time-invariant scheduler motivated by the observation that input process to a transmitter is correlated over time. Recent traffic measurements point out the presence of long range dependence (LRD) and self-similarity for packets on networks [9], packets from variable bit rate (VBR) video [1]. The input arrivals are not independently distributed, as assumed in most current studies. We demonstrate that the input correlation plays an important role in determining the optimal scheduling policy.

Second, we reveal a relationship between reliable transmission rate, referred to as capacity, and Quality of Service (QoS) requirement, in terms of deadline miss rate. Existing energy optimized scheduling policies without future knowledge of input arrivals save energy by transmitting packets slowly. They may lead to unexpected deadline misses. By investigating the relation between capacity and deadline misses, we provide a stochastic QoS guarantee characterized by a deadline miss rate.

The rest of the paper is organized as follows. Section II reviews related work. In Section III, we provide problem formulation and a scheduling policy that takes input correlation structure into account. Relationship of reliable transmission rate and QoS requirement is revealed in Section IV. Section V verifies the analysis results through simulation. Section VI concludes the article.

## II. RELATED WORK

Different approaches have been proposed to investigate the tradeoff between power and delay. Average queuing delay was considered in [3], [2], [10]. Collins and Cruz proposed optimal transmission scheme in a fading channel with average delay constraint and a peak transmitter power [3]. They used a simplistic channel model with a two-state Markov chain and assumed that energy expenditure is linear with transmitted data. Berry and Gallager considered the energy minimization

problem with average buffer delay in a block-fading channel [2]. The energy minimization was turned into a convex optimization problem and dynamic programming was used to find the optimal solution. Rajan *et al.* studied the energy gain due to source burstiness with mean queueing delay constraints [10]. An optimal scheduler obtained by dynamic programming and a near-optimal approximation were proposed for energy savings over both Gaussian and fading channels. They assumed independent and identically distributed (i.i.d.) packet arrivals at each time slot. They pointed out that i.i.d. source models rarely represent real sources, which usually have strong time-correlation. However, they focused on the utility of energy-aware packet scheduling in a real system and did not study the impact of the correlation.

A different delay constraint was applied in [12], [6] in which a deadline was put to all packets. The indirect bound on packet delay requires all packets arrive before  $T$  to be transmitted no later than  $T$ . Off-line optimal and on-line near-optimal algorithms were proposed for a single transmitter-receiver pair by Uysal-Biyikoglu *et al.* [12]. They also considered the problem of scheduling packets over an infinite time  $T$  under a stability guarantee. They showed that their proposed scheduler can save more energy than a deterministic constant service policy. Same delay constraint was used in their later extension to multiple users [6].

Nuggehalli *et al.* [8] utilized both average delay and deadline constraints considering the effect of energy recovery during idle periods. They first adapted the scheduling policy in [12] with a battery recovery model and reported more than 50% energy saving. They then applied average delay and derived optimal solution with and without battery recovery. They again showed that the battery aware policy could save substantial energy.

More recently, explicit deadline for each packet was considered for delay-sensitive applications. Fu [5] proposed a packet transmission policy to send an amount of data within a fixed time period. Their focus was on the impact of fading channel on throughput and energy optimization. Khojastepour and Sabharwal considered strict maximum delay constraint for each arrived packet [7]. They established the connection between maximum delay scheduling and a low-pass filter. Two optimal scheduling approaches were proposed. One is time-invariant for i.i.d. input. The other is time-variant in which the scheduling adapts in response to input arrivals. Although the time-variant scheduler gives a lower energy consumption, the time-invariant scheduler is still useful because of its ease of computation. In this paper, we extend the results for time-invariant scheduler by considering input correlation structure and investigate the impact of scheduling on QoS.

The time-invariant scheduler was also extended by Zafer and Modiano [14]. The authors proposed an off-line generalization of the energy minimization problem and an on-line scheduler for a Poisson arrival. The on-line solution can be interpreted as the time-invariant scheduler plus anticipation of future arrivals. The scheduler can lead to less energy expenditure. However, they assumed that the input arrivals at different time were i.i.d.

and the arrivals followed a Poisson process, which is not always valid.

### III. DELAY-CONSTRAINED SCHEDULING

In this section, we first present a formulation of the problem. Then we give a brief review of existing results for independent input arrivals and extend the scheduler to support correlated input.

#### A. Problem Formulation

We consider a pair of transmitter and receiver. The transmitter is associated with an input buffer. The number of input packets to the buffer is given by a random process  $X_t$ . The size of each packet is assumed to be long enough for reliable communication close to the mutual information of the channel. The arriving packets are buffered in the input queue before transmission. Each packet is associated with a delay constraint  $D$ .

The transmitter schedules packets out of the queue at a rate of  $R_t$  at time  $t$  and uses power  $P_t$  for transmission. We use a causal scheduler without future knowledge of packet arrivals. Consider an additive white Gaussian channel between the transmitter and receiver. The energy function is monotonically increasing and strictly convex in  $R_t$ . For example, it was shown that the average power is exponentially related to the rate, i.e.  $P_t \propto 2^{R_t}$  [7]. The objective of this study is to schedule packets and adapt transmission rate so that the average energy consumption is minimized while retaining the timing constraints of each packet. Formally, it is to minimize

$$\mathbb{E}[2^{R_t}] = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{t=0}^N 2^{R_t}, \quad (1)$$

subject to the timing constraints.

#### B. Independent Input Arrivals

According to [7], the output transmission rate can be modeled as

$$R_t = h(0)X_t + h(1)X_{t-1} + \dots + h(D-1)X_{t-D+1}, \quad (2)$$

where  $0 \leq h(i) \leq 1$  and  $\sum_{i=0}^{D-1} h(i) = 1$ . The equation represents the scheduling process as a perfect form of linear system model with a scheduling function  $h(t)$ .

Based on the linear property of the scheduler, the authors proposed an optimal scheduler independent of input arrival distribution. When the input arrivals over different time are i.i.d, they proved that the coefficients are uniformly distributed and each is equal to  $1/D$ . The transmission rate becomes

$$R_t = \frac{X_t + X_{t-1} + \dots + X_{t-D+1}}{D}. \quad (3)$$

#### C. Correlated Input Arrivals

Consider a time interval  $I = [t_1, t_2]$ . The average transmission rate in the interval is  $\mu_r = \sum R_t / (t_2 - t_1)$ , where the sum is taken over each packet  $i$  with  $[t_i, t_i + D] \subseteq [t_1, t_2]$ . Clearly,  $\mu_r$  is a lower bound on the average transmission rate,

which must be achieved by any scheduler over the interval  $I$ . By the convexity of the objective function (1), a constant transmission rate of  $\mu_r$  in  $[t_1, t_2]$  is optimal in the sense that no feasible schedule can use less power. However, because of the variance of input arrivals, a constant speed setting in the interval may not guarantee all packets scheduled before their deadlines. For example, there may exist an input arrival at time  $t_i$  which requires a higher rate than  $\mu_r$  to be transmitted before  $t_i + D$ . In this case, a constant rate will lead to deadline misses. Considering the delay constraint, we expect an energy-efficient feasible scheduler to assign the transmission rate as close as possible to the constant rate  $\mu_r$ .

We measure the difference between the resulted rate  $R_t$  and the constant rate as  $R_t - \mu_r$ . To find a schedule that is closest to the constant, we apply a commonly used criterion, the minimum of the mean square error (MSE),  $\mathbb{E}[(R_t - \mu_r)^2]$ . The objective is to find the a scheduling function  $h(t)$  so that the MSE is minimized.

Consider the input process that is wide sense stationary (WSS)—the mean of  $X_t$  is constant and its autocorrelation function depends only on the time difference. We note that the assumption of WSS is general in modeling self-similar traffic. For example, traffic traces generated from synthetic Fractional Gaussian Noise processes (FGN) are WSS. We formulate the optimization problem in Theorem 3.1.

*Theorem 3.1:* The optimal delay bounded scheduling has one and only solution.

*Proof sketch:* Define  $\mathbf{h} = [h(0), h(1), \dots, h(D-1)]'$  and  $\mathbf{X}_t = [X_t, X_{t-1}, \dots, X_{t-D+1}]$  as vector forms of the scheduling policy and packet arrivals. Write (2) in vector format as  $R_t = \mathbf{h}'\mathbf{X}_t$ . It follows that the mean transmission rate is

$$\mu_r = \mathbb{E}[R_t] = \mathbb{E}[\mathbf{h}'\mathbf{X}_t] = \mathbb{E}[X_t].$$

The explanation is that the transmitter should transmit all packets, because we assume no degradation in the transmission (sum of  $h(i)$  is one).

The optimization problem is to minimize

$$\begin{aligned} & \mathbb{E}[(R_t - \mu_r)^2] = \mathbb{E}[R_t^2] - \mu_r^2 \\ & = \mathbb{E}[\mathbf{h}'\mathbf{X}_t\mathbf{h}'\mathbf{X}_t] - (\mathbb{E}[X_t])^2 = \mathbb{E}[\mathbf{h}'\mathbf{X}_t\mathbf{X}_t'\mathbf{h}] - (\mathbb{E}[X_t])^2 \\ & = \mathbf{h}'\Omega\mathbf{h} - (\mathbb{E}[X_t])^2, \end{aligned} \quad (4)$$

where  $\Omega = \mathbb{E}[\mathbf{X}_t\mathbf{X}_t']$ , being the covariance matrix of input process  $X_t$  in the order of  $D$ .

As the input process is WSS,  $\mathbb{E}[X_t]$  is a constant. The optimization problem is to minimize  $\mathbf{h}'\Omega\mathbf{h}$ . In practice, it is impossible to determine one component of  $\mathbf{X}_t$  from another [4]. This means the correlation matrix is strictly positive definite. There exists an orthogonal matrix  $\mathbf{U}$ , such that  $\mathbf{U}^{-1}\Omega\mathbf{U} = \Lambda$  and  $\Lambda = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_D)$  where  $\lambda_i$  are the eigenvalues of the matrix  $\Omega$  and  $\lambda_i > 0$ .

Let  $y = \mathbf{h}'\Omega\mathbf{h}$ . It follows that  $y = \mathbf{h}'\mathbf{U}\Lambda\mathbf{U}^{-1}\mathbf{h}$ . Define  $\mathbf{g} = \mathbf{U}^{-1}\mathbf{h}$ . It follows  $y = \mathbf{g}'\Lambda\mathbf{g}$ . This is a standard form. It is minimized when  $\lambda_i g_i^2 = \lambda_j g_j^2$  for any  $1 \leq i, j \leq D$ . Since

$\mathbf{h}'\mathbf{1} = 1$  and  $\mathbf{U}\mathbf{g} = \mathbf{h}$ , there exists one and only one solution of  $\mathbf{h}$  for the optimization problem.  $\square$

This theorem reveals the impact of input correlation structure on scheduling. The structure can be either measured dynamically or assumed according to past packet arrivals. A special case is when the input arrivals are independent. The covariance matrix becomes diagonal, and the solution becomes uniform, which means equal amount of transmission is scheduled before the deadline. In this case, the resulted scheduling policy is the same as that in [7].

#### IV. QOS GUARANTEE

Practical communication systems are usually limited with a highest reliable transmission rate, referred to as capacity. An *outage* event occurs when the attempted transmission rate is higher than the capacity. The energy-efficient scheduling policies represented by (2) try to postpone packet transmission as late as possible subject to the delay constraint. More outages are possible by transmitting with a lower rate.

For example, consider a system with transmission capacity 100 bytes per scheduling epoch (or time slot). We have two arrivals from an independent input. One arrives at time 1 with a size of 100 bytes; the other at time 2 with a size of 400 bytes. Both have a delay constraint 4. If we do not consider energy consumption and schedule transmission as fast as possible, the first input will be scheduled at time 1 and the second will be scheduled from time 2 to 5, all with 100B per slot. It is a feasible scheduling and no outage will occur. For the optimal time-invariant scheduler, according to (3), the transmission for each input will be scheduled uniformly, with 1/4 per slot. Thus for the first input, 25 bytes will be scheduled from time 1 to 4. For the second, 100 bytes will be scheduled from time 2 to 5. We can see that from time slot 2 to 4, the scheduled rate will be 125B per slot, which results in outages. Since no future packet arrivals are assumed, outages can not be avoided using the power optimized scheduler.

To guarantee zero-outage (error-free) packet transmission, we postpone the transmission of a packet that will result in an outage. In the case, a packet may fail to be transmitted before its deadline which we refer to as a deadline miss or an overload. To restrict deadline misses in a controllable manner, we next investigate the relationship between reliable transmission rate and a level of QoS requirement, characterized by a deadline miss rate or, alternatively, an overload probability.

If the input distribution of the transmission rate is unimodal, meaning that for some mode  $m$ , its CDF is concave on  $[m, \infty)$  and convex on  $(-\infty, m]$ . Many widely used standard distributions are unimodal, for examples, Gaussian and Poisson distributions. Define the overload probability  $v$  as  $\text{prob}(R_t > c)$ , where  $c$  is the maximum reliable transmission rate. With known mean  $\mu_r$  and standard deviation  $\sigma_r$  of the transmission rate  $R_t$ , we can estimate the output probability tail distribution.

*Theorem 4.1:* The upper capacity bound for transmission

rate with a unimodal distribution can be expressed as

$$v \leq \frac{4\sigma_r^2}{4\sigma_r^2 + 9(\mathbf{c} - \mu_r)^2}. \quad (5)$$

*Proof sketch:* From Vysochanskiĭ and Petunin inequality [13], we can derive the probability that  $R_t$  exceeds capacity  $\mathbf{c}$  as:

$$P(R_t \geq \mathbf{c}) \leq \begin{cases} \frac{4E[R_t^2]}{4E[R_t^2] + 9\mathbf{c}^2} & \text{if } \mathbf{c}^2 \geq \frac{8}{3}E[R_t^2], \\ \frac{4E[R_t^2]}{3(4E[R_t^2] + 9\mathbf{c}^2)} - \frac{1}{3} & \text{if } \mathbf{c}^2 < \frac{8}{3}E[R_t^2]. \end{cases}$$

In practice, we only need to consider the first constraint  $\mathbf{c}^2 \geq \frac{8}{3}E[R_t^2]$ , because the second constraint will lead to high deadline misses. We have

$$P(R_t \geq \mathbf{c}) = P(R_t - \mu_r \geq \mathbf{c} - \mu_r) \leq \frac{4\sigma_r^2}{4\sigma_r^2 + 9(\mathbf{c} - \mu_r)^2},$$

if  $\mathbf{c} \geq \sqrt{\frac{5}{3}}\sigma_r + \mu_r$ .  $P(R_t \geq \mathbf{c})$  equals  $v$ . This completes the proof.  $\square$

The theorem reveals that capacity is determined by the transmission rate mean and variance. Because the high variability of input arrivals, the transmission rate variance can be a dominating factor in determining the capacity.

With a given capacity, we can determine the delay constraint subject to a QoS requirement by the following corollary.

*Corollary 4.1:* If the input arrivals are independent over time  $t$ , with  $\mu_x$  and  $\sigma_x$  as the mean and standard deviation of input distribution. The relationship of deadline, capacity, chance of failure, and input statistics can be characterized by equation

$$D = \frac{4(1-v)\sigma_x^2}{9v(\mathbf{c} - \mu_x)^2}. \quad (6)$$

**Proof:** For independent input, the scheduling policy should be set to a constant value  $1/D$  for period  $0 \leq t < D$  for minimizing transmission energy consumption. Thus,

$$\sigma_r^2 = \sigma_x^2 \sum_{t=-\infty}^{t=\infty} h_t^2 = \frac{\sigma_x^2}{D}.$$

From Theorem 4.1, we can expression the capacity  $\mathbf{c}$  as

$$\mathbf{c} = \frac{2}{3} \sqrt{\frac{1-v}{v}} \cdot \sigma_r + \mu_r.$$

Combining the two equations completes the proof.  $\square$

This corollary shows that the delay constraint can be adapted in response to the change of traffic intensity. To keep the same level of QoS for traffic with different means and variances, say  $(\mu_1, \sigma_1^2)$  and  $(\mu_2, \sigma_2^2)$ , the ratio of their deadlines is

$$\frac{D_2}{D_1} = \frac{\sigma_2^2 \cdot (\mathbf{c} - \mu_1)^2}{\sigma_1^2 \cdot (\mathbf{c} - \mu_2)^2}. \quad (7)$$

When  $\mathbf{c} \gg \mu_1$  and  $\mathbf{c} \gg \mu_2$  or  $\mu_1 = \mu_2$ , the ratio can be expressed as  $\sigma_2^2/\sigma_1^2$ .

## V. PERFORMANCE EVALUATION

We conducted simulations to verify the analytical results. They were carried out in three aspects: (1) Compare the

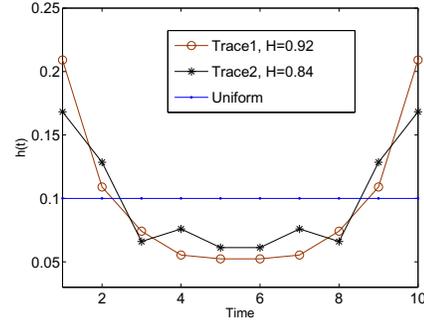


Fig. 1. Comparison of scheduling functions with different input correlation.

proposed scheduling policy with existing approach for correlated input; (2) Investigate the effectiveness of the capacity bounds; (3) Demonstrate the application of the capacity-QoS relationship for QoS control.

The first experiment was conducted using video traces from [11]. An important characteristic of the traces is the time correlation. To show the performance of the schedulers under traffic with different degree of correlation, we chose two VBR video traces, JurassicPark I (Trace1) and Simpsons (Trace2), with different Hurst parameters ( $H$ ). The Hurst parameter is used to characterize the long range dependence (LRD) of the traffic. The degree of correlation is high with a large  $H$ . The Hurst parameters are 0.92 and 0.84 for JurassicPark and Simpsons respectively. Their impacts on the scheduling function  $h(t)$  are shown in Figure 1. The values are obtained by solving (4). The uniform allocation from [7] is included for reference. The scheduling functions for both traces differ from the uniform allocation. The difference increases with the degree of correlation. This can be verified from the low-pass nature of the scheduling process. With increased degree of LRD, there are more low frequency components [1]. To effectively smooth the input, the low-pass filter needs to have a lower bandwidth. The filter for the trace JurassicPark has the lowest bandwidth, which is most effective in getting a smoothed output.

To compare the energy consumption of the proposed scheduling policy with the uniform allocation, we simulated transmission of the video packets. Figure 2 shows the energy consumptions with different packets. We assumed packets had the same delay constraint for simplicity. The energy consumptions were normalized with respect to a schedule that transmits packets as fast as possible without energy consideration. It can be observed that both schedulers are effective in energy savings. The proposed scheduler determined by input correlation consistently outperforms the uniform allocation. As the delay constraints increase, the energy savings by both schedulers increase. This is because the transmission rate is reduced with relaxed delay constraints. Due to the convexity of the power function, more energy is saved. Note that the trace JurassicPark benefits more from the consideration of input correlation. This is expected because high input correlation makes a uniform allocation less effective for energy optimization. It can also be explained by the difference of scheduling functions

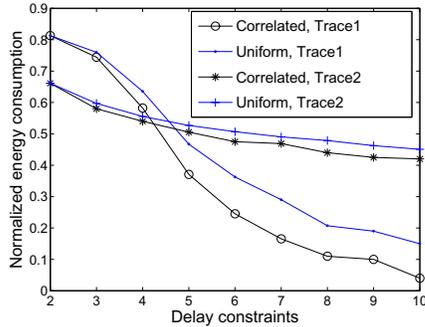


Fig. 2. Energy consumption with different delay constraints.

TABLE I

SCHEDULING WITH INCREASED DELAY OR CAPACITY FOR  $n(200, 60)$ .

Scheduling	$c = 249, D = 23$
Trans. Rate Mean	200.4
Trans. Rate Var.	156.5
Completion Time Mean	22.7
Completion Time Var.	1.9
Deadline Miss Rate	0.8%

revealed in the first experiment.

With a given QoS level, we can get the relation between capacity and delay by Corollary 4.1. Interesting questions are if there are more input arrivals than expected, to sustain the same level of QoS, how much degradation there should be and what is the capacity required to keep the same delay constraint. Following the previous experiment with more variable input arrivals coming in normal distribution  $n(200, 60)$ , we can calculate the new delay constraint  $D_{new}$  based on the constraint  $D_{old}$  for  $n(200, 40)$  as:

$$D_{new} = \frac{\sigma_{new}^2}{\sigma_{old}^2} \times D_{old} = \frac{60^2}{40^2} \times 10 \approx 23.$$

We ran simulations with a new deadline of 23 time units. The capacity was set to 249 with a target deadline miss rate 1%, an upper bound calculated by using input distribution information. The results are shown Table I. We observe that in response to the change of incoming traffic, about 99.2% of the packets can meet their adjusted deadline of 23.

If we want to keep the original deadline of 10 time units, a higher reliable transmission rate is needed. According to (7), we have

$$\frac{\sigma_{new}^2 \cdot (c_{old} - \mu_{old})^2}{\sigma_{old}^2 \cdot (c_{new} - \mu_{new})^2} = 1.$$

As a result, the new capacity  $c_{new} = 274$ . We ran simulations based on this new predicted capacity. With the adjusted capacity, 1.1% of the packets miss their deadlines, close to the target QoS level.

## VI. CONCLUSION

Energy-efficient packet transmission is an important aspect in wireless communication. We consider minimal power transmission over Gaussian channels with delay constraints for individual packet. Existing approach either works only for a certain type of input with known marginal distribution of the input process or assumes the input is independent. In light of the fact that the input process for a wireless transmitter is often correlated over time and may not follow any distributions, we consider the impact of input correlation structure on scheduling without a priori knowledge of input distribution. An optimal scheduling policy has been derived for different input correlation patterns, in which the independent input is only a special case. We then reveal the relationship between the maximum reliable transmission rate and a level QoS of requirement, characterized by a deadline miss rate. The relationship holds for all types of unimodal input distributions with finite first and second order moments. Simulation results validate the relationships and demonstrate the effectiveness of the proposed scheduling policy in energy savings.

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## REFERENCES

- [1] J. Beran, R. Sherman, M. Taqqu, and W. Willinger. Long-range dependence in variable bit-rate video traffic. *IEEE/ACM Transactions on Communications*, 43(234):1566–1579, Feb./Mar./Apr. 1995.
- [2] R. A. Berry and R. G. Gallager. Communication over fading channels with delay constraints. *IEEE Transactions on Information Theory*, 48(5):1135–1149, 2002.
- [3] B. Collins and R. L. Cruz. Transmission policies for time varying channels with average delay constraints. In *Proceedings of Allerton Conference on Communication, Control, and Computing*, 1999.
- [4] W. Feller. *An Introduction to Probability Theory and Its Applications (Vol II)*. John Wiley & Sons, Inc, 1971.
- [5] A. Fu, E. Modiano, and J. N. Tsitsiklis. Optimal energy allocation for delay-constrained data transmission over a time-varying channel. In *Proceedings of the IEEE Infocom*, 2003.
- [6] A. E. Gamal, C. Nair, B. Prabhakar, E. Uysal-Biyikoglu, and S. Zahedi. Energy-efficient scheduling of packet transmissions over wireless networks. In *Proceedings of the IEEE Infocom*, 2002.
- [7] M. A. Khojastepour and A. Sabharwal. Delay-constrained scheduling: Power efficiency, filter design, and bounds. In *Proceedings of the IEEE Infocom*, March 7-11 2004.
- [8] P. Nuggehalli, V. Srinivasan, and R. R. Rao. Delay constrained energy efficient transmission strategies for wireless devices. In *Proceedings of the IEEE Infocom*, 2002.
- [9] V. Paxson and S. Floyd. Wide-area traffic: The failure of poisson modeling. *IEEE/ACM Transactions on Networking*, 3(3):226–244, 1995.
- [10] D. Rajan, A. Sabharwal, and B. Aazhang. Delay-bounded packet scheduling of bursty traffic over wireless channels. *IEEE Transactions on Information Theory*, 50(1):125–144, 2004.
- [11] P. Seeling, M. Reisslein, and B. Kulapala. Network performance evaluation using frame size and quality traces of single-layer and two-layer video: A tutorial. *IEEE Comm. Surveys and Tutorials*, 6(2):58–78, 2004.
- [12] E. Uysal-Biyikoglu, B. Prabhakar, and A. E. Gamal. Energy-efficient packet transmission over a wireless link. *Transactions on Networking*, 10(4):487–499, 2002.
- [13] D. Vysochanskii and Y. Petunin. Justification of the three-sigma rule for unimodal distributions. *Theory of Probability and Mathematical Statistics*, 21:25–36, 1980.
- [14] M. Zafer and E. Modiano. A calculus approach to minimum energy transmission policies with quality of service guarantees. In *Proceedings of the IEEE Infocom*, 2005.